

Answer the following questions:

Q1. a) Find the points, if any, on the graph of

$$y^2 - 6x^2 + 4x + 1 = 0$$

at which the tangent lines have slope 4.

(8 marks)

Q2. a) Find $\frac{d}{dx} \sqrt{\frac{x^2+1}{x^3+x^2+2}}$.

(5 marks)

b) Find $\lim_{x \rightarrow 1} \frac{\sqrt{x+3} - 2}{x-1}$.

(5 marks)

c) Suppose f is a function continuous on $[a,b]$, differentiable on (a,b) , and satisfies $f(a) < f(b)$. Show that there exists a value $c \in (a,b)$ such that $f'(c)$ is positive.

(5 marks)

Q3. a) The radius r of a cylinder decreases at a rate of 4 m/sec. and the altitude h increases at a rate of 2 m/sec. Does the volume increase or decrease when

$r = 5$ m and $h = 4$ m? Hint: Volume = $\pi r^2 h$.

(10 marks)

b) Use differentials to approximate the value of $\sqrt{3.91}$.

(6 marks)

Q4. Let $f(x) = (x^2 - 1)^{2/5}$.

a) Find the intervals where f is increasing or decreasing.

b) Find the intervals where f is concave upward or downward.

c) Find the local extreme values of f .

d) Sketch the graph of f .

(20 marks)